

Prob. num.3583, Crux Mathematicorum with Mathematica Myhem
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Let α and β be nonnegative real numbers and define

$$a_n = (n + \ln(n+1)) \prod_{k=1}^n \frac{\alpha + k + \ln k}{\beta + (k+1) + \ln(k+1)}$$

,

$$p_n = (\alpha + n + 1 + \ln(n+1)) \prod_{k=1}^n \frac{\alpha + k + \ln k}{\beta + (k+1) + \ln(k+1)}$$

Find those nonnegative real numbers α and β for which $\sum_{n=1}^{\infty} a_n$ converges and determine the relation between α and β that ensures that

$$\sum_{n=1}^{+\infty} \left(a_n - p_n \ln \left(1 + \frac{1}{n+1} \right) \right) = (\alpha + 1)(\alpha + 2 + \ln 2) - \frac{(\alpha + 1)^2}{2}$$